



Semester One Examination, 2025

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

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In words

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Circle your Teacher's Name: Ms Lee Mr Liu Mr Mohammadi Mrs Murray

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	90	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
8	3	
9	10	
10	7	
11	8	
12	6	
13	9	
14	8	
15	7	
16	8	
17	9	
18	7	
19	8	
Section 2 Total	90	
Weighted ($\times 0.6915$)	65%	

Section Two: Calculator-assumed**65% (90 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8**(3 marks)**

Solve the following equation

$${}^xP_2 + {}^xP_3 = 3x^2 + 7x$$

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Question 9

(10 marks)

Consider the vectors $\vec{a} = \begin{pmatrix} 4\lambda + 3 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 3\lambda - 4 \\ 1 \end{pmatrix}$, where λ is a constant.

(a) When $\lambda = -1$ determine

(i) the exact unit vector $\hat{r}$ when $\vec{r} = \vec{a} - \vec{b}$. (2 marks)

(ii) the angle between $\vec{a}$ and $\vec{b}$ to the nearest 0.1 of a degree. (2 marks)

(iii) the vector projection of $\vec{b}$ on $\vec{a}$. (2 marks)

(b) Determine the exact value(s) of λ when $\vec{a}$ and $\vec{b}$ are

(i) parallel. (2 marks)

(ii) perpendicular. (2 marks)

Question 10

(7 marks)

- (a) A teacher wrote eight distinct positive integers on the board. None of these numbers had a prime factor larger than 15.
- (i) Prove that there are two integers on the board that have the same prime factor. (2 marks)
- (ii) The teacher erased one of the eight integers on the board. Explain why it is now impossible to prove that there are two integers on the board that have the same prime factor. (2 marks)
- (b) A group of 60 students was asked about their preferences for ice cream flavours. 28 liked chocolate, 22 liked vanilla and 25 liked strawberry. 5 liked all three flavours. 12 liked chocolate and vanilla, 9 liked strawberry and chocolate, and 8 liked vanilla and strawberry. **Clearly showing use of the inclusion-exclusion principle**, find the probability that a student chosen at random from this group liked only one flavour of ice cream. (3 marks)

Question 11

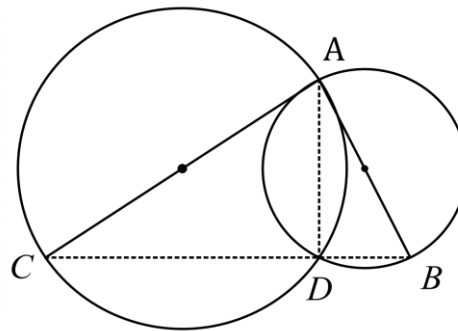
(8 marks)

- (a) The statement $\forall \alpha, \beta \in \mathbb{R}; \sin \alpha = \sin \beta \Rightarrow \alpha = \beta$ is written in the formal language of proof.
- (i) Write the statement in words as if you were reading it to an audience. (2 marks)
- (ii) Briefly explain whether the statement is true or false. (1 mark)
- (b) Consider the following conditional statement about polygons with no crossed edges: If it is a quadrilateral, then the sum of its interior angles is 360° .
- (i) Write the contrapositive statement. (1 mark)
- (ii) Write the converse statement. (1 mark)
- (c) Let n be an integer. Prove that $\frac{n^3 - n}{6}$ is always an integer. (3 marks)

Question 12

(6 marks)

Two circles intersect at points A and D so that their centres lie outside the other circle. The diameter AB of one circle is less than the diameter AC of the other circle, as shown in the diagram.



- (a) Prove that D lies on the straight line from B to C .

(3 marks)

Point E lies on minor arc AD of the larger circle and $\angle CAD = 62^\circ$.

- (b) Determine, with justification, the size of $\angle AED$.

(3 marks)

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Question 13**(9 marks)**

Forces $\vec{b}$ and $\vec{d}$ act on a small body. Force $\vec{b}$ has magnitude 175 newtons and acts on a bearing of 307° . Force $\vec{d}$ has magnitude 115 newtons and acts on a bearing of 218° .

- (a) Sketch a labelled triangle to show the relationship between the two forces and their resultant $\vec{r}$, and show a known angle in the triangle. (2 marks)

- (b) Determine the magnitude of $\vec{r}$ and the bearing on which it acts. (4 marks)

The work done, in joules, by a force moving a body is the scalar product of the force acting on the body, in newtons, and the displacement of the body, in metres.

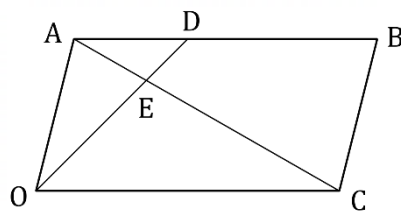
- (c) Determine the work done by the resultant of $\vec{b}$ and $\vec{d}$ when it causes the body to move 214 centimetres on a bearing of 291° . (3 marks)

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Question 14

(8 marks)

$OABC$ is a parallelogram with $\overrightarrow{OA} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.



(a) Determine $\overrightarrow{OB}$.

(1 mark)

(b) Determine $\overrightarrow{OD}$ given that D is a point on AB such that $3\overrightarrow{AD} = \overrightarrow{AB}$.

(2 marks)

OD and AC intersect at point E such that $\overrightarrow{AE} = \lambda\overrightarrow{AC}$.

(c) Show that $\overrightarrow{OE} = (1 - \lambda)\overrightarrow{OA} + \lambda\overrightarrow{OC}$.

(2 marks)

(d) Determine the value of λ .

(3 marks)

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Question 15**(7 marks)**

Let M be the midpoint of chord AB in a circle with centre O , so that M and O are not coincident.

- (a) Let $\underline{a} = \overrightarrow{OA}$ and $\underline{b} = \overrightarrow{OB}$. Use a vector method to prove that OM is perpendicular to AB .

(4 marks)

OM is extended to cut the minor arc AB at C and $\angle OAM = 24^\circ$.

- (b) Determine, with justification, the size of $\angle CBM$.

(3 marks)

Question 16

(8 marks)

- (a) Players in a club are assigned to the junior or senior squad, but not both. Determine the number of distinct ways in which a coach can select 12 players when 3 must be chosen from the junior squad of 8, and the remainder from the senior squad of 11. (2 marks)
- (b) As part of a training exercise, a squad of 16 players must be divided into three groups, one with 7 players, one with 5 players and the other with 4 players. Determine the number of distinct ways in which the division can be made. (2 marks)
- (c) When r distinct objects are divided into groups of x, y and z objects, where $x + y + z = r$, prove that $\binom{r}{x} \binom{y+z}{z} = \binom{r}{z} \binom{y+x}{x}$. (4 marks)

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Question 17

(9 marks)

A small model boat leaves buoy A with constant velocity $\begin{pmatrix} 4 \\ -31 \end{pmatrix}$ centimetres per second, and t seconds later it reaches buoy B . Buoys A and B are moored in a river and have position vectors $\begin{pmatrix} 31 \\ 455 \end{pmatrix}$ and $\begin{pmatrix} 127 \\ -217 \end{pmatrix}$ centimetres respectively.

A steady current is flowing in the river with velocity $\begin{pmatrix} m \\ n \end{pmatrix}$ centimetres per second.

(a) Show that the constants m and n are related by the equation $n = 3 - 7m$. (4 marks)

(b) Given that the speed of the current is $\sqrt{17}$ centimetres per second and that $n > m$, determine the velocity vector of the current. (3 marks)

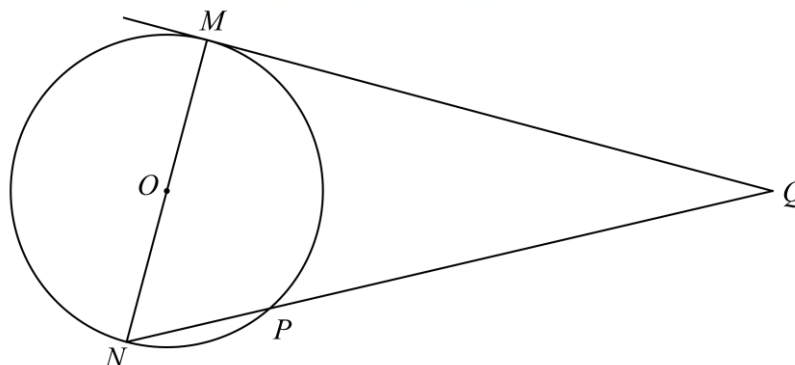
(c) Determine how long the boat takes to travel from A to B . (2 marks)

Question 18**(7 marks)**

- (a) Passwords are to be formed using all 8 symbols in the set $\{*, !, =, +, ^, \%, \#, \&\}$ without repetition, such as $= !\% + \&^{\#} *$. Braces $\{\}$, full stops and commas are not classified as symbols here. Determine how many different passwords can be made
- (i) with no other restrictions. (1 mark)
- (ii) that start with $\#$ or end with $\%$. (3 marks)
- (b) Three-digit numbers are to be formed using permutations of three digits from the set $\{3, 4, 5, 6, 7, 8, 9\}$ without repetition. Determine how many even numbers less than 680 can be formed in this way. (3 marks)

Question 19**(8 marks)**

In the diagram, MQ is tangential to the circle with centre O and diameter MN at M . Line NQ cuts the circle at P .



- (a) The tangent to the circle at P cuts MQ at R . Show this on the diagram, draw chord MP and let $\angle PMR = x^\circ$. (1 mark)
- (b) Prove that $\angle RPQ = 90^\circ - x^\circ$. (4 marks)

- (c) Prove that $RM = RQ$. (3 marks)

Supplementary page

Question number: _____

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